Methods and Algorithms of Freak Waves Detection in the Coastal Zone

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Abstract

Methods of forecasting and detection of freak waves in computational experiments are considered. Operational forecasting methods of the freak waves, remote sensing of the sea surface are constructed and analysed. The application of numerical experiments to verify the in-situ measurements were considered.

Introduction

Freak waves occupy a special place among the catastrophic events in the ocean. These solitary waves occur suddenly and can reach large amplitude (known cases of up to 30 m). The suddenness of these waves determines their risk for ships and offshore
structures. Problem of prediction and detection of freak waves in the ocean is one of the most relevant in the study of freak waves. Recently, the study of freak waves has made significant progress via computational experiments (Kharif et al., 2009; Henderson et al., 1999; Bateman et al., 2001; Chalikov, 2009; Dyachenko and Zakharov, 2008; Zakharov and Shamin, 2010; Boukhanovsky et al., 2005; Lopatoukhin and Boukhanovsky, 2004; Divinskii et al., 2004; Zaitsev et al., 2011).

We consider such important issues as the detection and prediction of occurrence of freak waves in the processing of numerical experiments. To solve this problem, we consider three approaches: an operational forecast based on the developed functional, remote monitoring of the dynamics of wave propagation and detection statistics comparing freak waves on the full record of the dynamics of waves and wave record.

The equations in conformal variables describing waves on water

In this paper, simulation of freak waves is based on the numerical solution of the equations describing the unsteady flow of an ideal fluid with a free surface. We consider a plane for an infinitely deep down. We consider a 2π-periodic conditions in horizontal variable. Such assumptions are natural for modelling freak waves.

Let ideal incompressible fluid occupies the region in the plane \((x, y)\) bounded by a free surface

\[-\infty < y < \eta(x, \tau), \ -\infty < x < +\infty, \ \tau > 0.\] (1)

Considering the motion of the fluid potential, we have \(v(x, y, \tau) = \nabla \Phi(x, y, \tau)\), where \(v(x, y, \tau)\) is two-dimensional velocity field, \(\Phi(x, y, \tau)\) is velocity potential. Throughout this paper we assume that the operator of the gradient, divergence and Laplace applied only to the space variables. From the condition of incompressible fluid, it follows that the velocity potential satisfies the Laplace equation

\[\Delta \Phi(x, y, \tau) = 0.\] (2)

With the following boundary and initial conditions:

\[\left(\eta_t + \Phi_x \eta_x - \Phi_y \right)_{y=\eta(x, \tau)} = 0,\] (3)

\[\left(\Phi_t + \frac{1}{2} |\nabla \Phi|^2 + g y\right)_{y=\eta(x, \tau)} = 0,\] (4)

\[\Phi_y \bigg|_{y=-\infty} = 0,\] (5)

\[\eta_t \big|_{t=0} = \eta_0(x), \quad \Phi_t \big|_{t=0} = \Phi_0(x, y).\] (6)

Here \(g\) is the acceleration of gravity.
We use these equations in conformal variables. The idea of using conformal variables to describe the unsteady flow of an ideal fluid with a free surface was firstly proposed in (Whitney, 1971; Ovsyannikov, 1973). For the numerical simulation of equations in conformal variables these idea used in (Dyachenko, 2001; Chalikov and Sheinin, 2005; Ruban, 2005) and many others. We consider the case of these equations proposed in (Zakharov et al., 2002).

We perform conformal mapping of the area occupied with a fluid to the bottom half-plane (with coordinates \( w = u + iv \)). This mapping is defined by the function \( z = z(w), \ z = x + iy \). The dynamical equations are formulated for variable

\[
R(t, w) = \frac{1}{z'(t, w)}, \quad V(t, w) = i \frac{\partial \Phi}{\partial z} \tag{7}
\]

and have following form

\[
\dot{R}(t, u) = i\left(U(t, u)R'(t, u) - U'(t, u)R(t, u)\right), \tag{8}
\]

\[
\dot{V}(t, u) = i\left(U(t, u)V'(t, u) - B'(t, u)R(t, u)\right) + g\left(R(t, u) - 1\right), \quad 0 < u < 2\pi, \quad 0 < t < T, \tag{9}
\]

\[
R(t, 0) = R(t, 2\pi), \quad V(t, 0) = V(t, 2\pi), \quad 0 < t < T, \tag{10}
\]

\[
R(0, u) = R_0(u), \quad V(0, u) = V_0(u), \quad 0 < u < 2\pi. \tag{11}
\]

Here, the functions \( U \) and \( B \) are calculated according to formulas:

\[
U = P\left(\bar{V}R + \bar{R}V\right), \quad B = P\left(\bar{V}V\right), \quad P = \frac{1}{2}(1 + iH). \tag{12}
\]

The mathematical correctness of the above equations was established in a series of papers (Shamin, 2006, 2007, 2008, a, b, 2010, 2011). The mathematical correctness of the above equations was established in a series of papers (Shamin, 2006, 2007, 2008, a, b, 2010, 2011). The existence and uniqueness of solutions of equations (8) – (11) have been established in these works, as well as efficient numerical methods have been proposed and convergence of numerical methods has been proven.

**Operational forecast of freak waves**

As in the computational experiments we observe the behaviour of the \( v(t) \) and \( \mu(t) \), it is possible to build an operational method of predicting the emergence of the freak waves on the basis of changes in these functions.

We have developed an algorithm SPRW (Simple Predictor of Rogue Waves), which is the following. The signal of predicting of occurrence of freak-wave issued when all the following conditions are met:
\[
\frac{v(t + \Delta) - v(t)}{\Delta} \geq \alpha, \quad \frac{\mu(t + \Delta) - \mu(t)}{\Delta} \geq \beta,
\]

where \( \Delta > 0, \alpha > 0, \beta > 0 \) is variables of the algorithm. On the basis of analysis of a large data bank of computational experiments were chosen the best parameters of the algorithm. Using this parameters algorithm SPRW showed prediction accuracy equal to 68.94 % taking into account the errors of the first and second kind. Type I error is when it will fail to test occurred freak wave, and the error of the second kind, on the contrary, there is a false positive test.

A typical situation is when the signal is issued in the immediate moment before the occurrence of freak wave that follows from the algorithm SPRW itself. From the experience of the application of the algorithm SPRW, built on analysing the behaviour of the functions \( v(t), \mu(t) \), we can conclude the stochastic behaviour of indicator functions. This conclusion is very much in line with the conclusions of (Kharif et al., 2009; Chalikov, 2009) about the absence of clear predictors of freak waves.

**Possibilities of remote detection of freak waves**

In the previous section we have considered the prediction of freak waves on the basis of analysis of solutions of differential equations describing the dynamics of water waves. However, a question about the possibility of remote detection of freak waves is important. We simulate a situation where field observations of waves is conducted with the help of an ideal point rangefinder. The main question addressed in this section is how to detect the emergence of freak waves on the basis of measurements of the "instrument".

So let fixed "measurer" is installed at a height \( H \). This measurer provides indications about the distance from the device to the surface of the waves along the line fixed at angle \( 0 < \alpha < \pi/2 \). This scheme is illustrated by Fig. 1.

Let a fixed time \( t \) we have the indication equal to \( R_t \), then the elevation of the surface at the point of intersection with the line above the zero level \( y_t \) expressed by the formula:

\[
y_t = H - R_t \sin \alpha.
\]

In the future, it is more convenient to work with variables \( y_t \). We will consider the results of computational experiments in which there are freak waves. A typical example of the results of the virtual measurer is shown in Fig. 2.

In this figure, we can determine the moment of the freak waves occurrence on the sharp increase in the graph. To implement the method of automatic detection of freak waves, we will work with function of the square average deviation of the function \( y_t \) from its mean value. We assume that our measurer indicates at discrete moments of time

\[
t_n = (\Delta t)n, \quad n = 0, 1, \ldots, N.
\]
Fig. 1: Scheme of measuring.

Fig. 2: Indications of virtual measurer. Freak wave was detected at \( t = 173 \).

We introduce following values

\[
\bar{y}_N = \frac{1}{N} \sum_{n=0}^{N} y_{t_n}; \quad \sigma_N^2 = \frac{1}{N-1} \sum_{n=0}^{N} (y_{t_n} - \bar{y}_N)^2.
\]  

(16)

where \( N \) is the number of samples processed. We can detect the moment of occurrence of freak waves, by analysing the change of \( \sigma_N^2 \) with increasing \( N \). Fig. 3 shows a graph of values \( \sigma_N^2 \) for computing experiment that was presented on Fig. 2.

It is necessary to determine the moments of sharp increase of \( \sigma_N^2 \) values to detect the occurrence of freak waves. That’s why, we analyse the behaviour of finite difference of magnitude \( \sigma_N^2 \).
\[ d_{N,\tau} = \sigma_{N,\tau}^2 - \sigma_{N-\tau}^2. \] (17)

Horizontal line in Fig. 4 is a critical value at which there is a registration freak waves. We should consider this indicator function, after the first 50 counts. The level of the critical line is set empirically. Computing experiments to detect the freak waves have shown its viability. It has been shown that the remote virtual measurer has the ability to detect the freak waves at a distance of 4-6 km.

The detection freak waves via records at the point

In field experiments, the analysis of marigram is often used. Marigram is a time-based recording surface elevation of the free surface at a fixed point. Through our experiments, we show that for wave record is possible to detect only a small portion of freak waves that occur in this area.

**Fig. 3:** Graph of values \( \sigma_{N}^2 \) for indications of virtual measurer.

**Fig. 4:** Graph of \( d_{N,\tau} \) values for \( \tau = 10 \).
In our computational experiments, we observe the dynamics of the free surface function. In our computational experiments, we observe the dynamics of the free surface function $y = y(x, t)$. On the basis of this function, we can detect extreme waves, using an amplitude criterion. And at the same time, we will build a marigram for this function, such as a measurer at the bottom.

We compare the results of the detection freak waves in computational experiments and in discrete marigram. We used marigram with sampling 1 Hz. In our experiments, we see that the percentage of coincidence of detection of freak waves in marigram and in full space record of waves rarely exceeds 1%. Thus, the detection of freak waves only wave record results in significantly distorted results. Thus using only marigrams for detection of freak waves leads to a significantly distorted results.

**Conclusion**

This article discusses methods for the detection and prediction of freak waves on the basis of computational experiments. The main conclusion is the occurrences of freak waves are stochastic in nature. This finding coincides with the conclusions of other authors. Therefore, for the construction of forecasting methods of occurrence freak waves necessary to use methods based on the theory of random processes and simulation. The results of the last section shows that the simulation of field experiments allows establishing that the study freak waves, based on the analysis of marigram leads to a significant distortion of the statistics freak waves.

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**References**


