About solvability and numerical simulation of nonstationary flow of ideal fluid with a free boundary

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Let an inviscid incompressible fluid occupy a domain in the plane (x,y) bounded by the free surface $-\infty < y \le \eta(x,t)$, $-\infty < x < \infty$, $\quad t > 0$. Assuming that the fluid flow is potential, we have $v(x,y,t) = \nabla\Phi(x,y,t)$, where $v(x,y,t)$ is a two-dimensional velocity field and $\Phi(x,y,t)$ is a potential.

The incompressibility condition $div\ v = 0$ implies that the velocity potential obeys the Poisson equation

$$\Delta\Phi(x,y,t) = 0. \tag{1}$$

Equation (1) is supplemented with the boundary and initial conditions

$$(\eta_t + \Phi_x\eta_x - \Phi_y)|_{y=\eta(x,t)} = 0,$$
$$(\Phi_t + \frac{1}{2}|\nabla\Phi|^2 + gy)|_{y=\eta(x,t)} = 0,$$
$$\Phi_y|_{y=-\infty} = 0,$$

where g is the acceleration of gravity.

We consider equivalent equations, called the Dyachenko's equations, describing nonstationary motion of ideal liquid with free boundary in a gravitational field. Dyachenko's equations are nonlinear integro-differential equations. They turn out to be convenient for numerical modeling.

Existence of analytic solutions of the above equations for a sufficiently small time interval is proved. Solutions from Sobolev spaces of finite order are also investigated.

In the second part of the work, a numerical method for obtaining approximate solutions is constructed. The convergence is proved, provided that a smooth solution exists. An efficient numerical scheme is proposed.

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[1] R. V. Shamin. On the Existence of Smooth Solutions to the Dyachenko Equations Governing Free-Surface Unsteady Ideal Fluid Flows Doklady Mathematics, 2006, Vol. 73, No. 1, pp. 112-113.